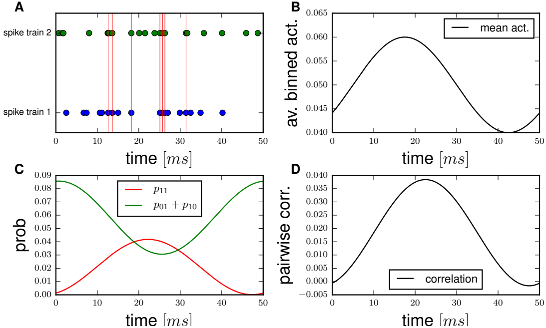
Correlations in binary networks with oscillating input

March 9, 2016 | Tobias Kühn[1], Michael Denker[1], PierGianLuca Mana[1], Sonja Grün[1,3] and Moritz Helias[1,2]
| [1]Institute of Neuroscience and Medicine (INM-6), Institute for Advanced Simulation (IAS-6), Jülich Research Centre and JARA, Jülich, Germany
[2]Department of Physics, Faculty I, RWTH Aachen University
[3]Theoretical Systems Neurobiology, Faculty I, RWTH Aachen University

Locking of surplus synchrony to LFP phase



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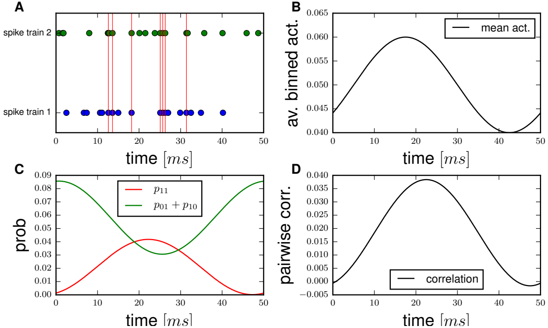

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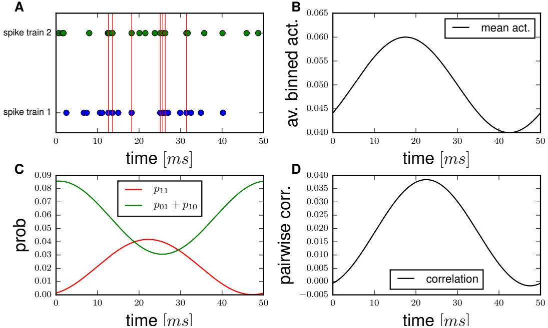

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- Do we observe modulation of simultaneous activity above chance level in a simple model system and if yes, what are the mechanisms?

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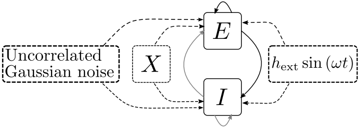

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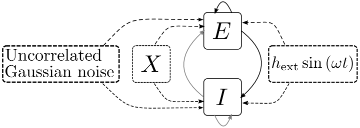

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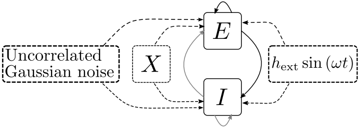

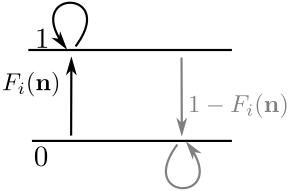

Figure: Helias et al., PLoS CB, 2014

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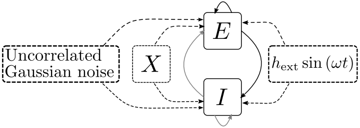

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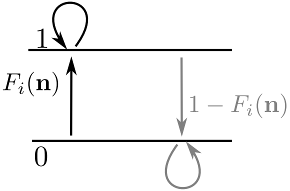

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- Glauber dynamics for asymmetric matrices:
- asynchronous updates
- after time *dt*, neuron n_i is chosen with prob $\frac{dt}{\tau}$ for an update, then in up-state with probability $F(\mathbf{n}/n_i)$.

Mean-field expressions for the first two moments

Master equation:

$$\underbrace{\tau}_{\text{update time}} \frac{\partial p}{\partial t}(\mathbf{n}, t) = A[p](\boldsymbol{n}) + B[p, F](\boldsymbol{n}), \ \forall \mathbf{n} \in \{0, 1\}^N,$$

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with a linear operator $A[\cdot](\boldsymbol{n})$, a bilinear operator $B[\cdot,\cdot](\boldsymbol{n})$ and (H being the Heaviside function)

$$F_i(\boldsymbol{n}, t) = H(h_i - \Theta), \ h_i = \sum_j J_{ij} n_j + h_{\text{ext}} \sin(\omega t)\,(+\text{noise}).$$

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- By multiplying the Master equation by $n_{i_1} n_{i_2} \cdot \ldots \cdot n_{i_K}$, moment-ODEs are derived (here for $K = 1, 2$).

ODE for mean activities and correlations

$$\tau \frac{d}{dt}\langle n_i(t)\rangle = -\langle n_i(t)\rangle + \langle F_i(\boldsymbol{n}, t)\rangle$$

$$\tau \frac{d}{dt}\langle n_i(t)\, n_j(t)\rangle = \left\{-\langle n_i(t)\, n_j(t)\rangle + \langle F_i(\boldsymbol{n}, t)\, n_j\rangle\right\} + \{i \leftrightarrow j\},$$

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- $F_i(\boldsymbol{n})$-terms involve moments of arbitrary order
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- $F_i(\boldsymbol{n})$-terms involve moments of arbitrary order
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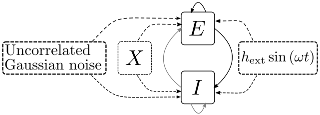
  (Ginzburg et al., PRE, 1994, Buice et al., Neur. Comp., 2010) .
- Gaussian closure: For evaluation of $F_i(\boldsymbol{n})$-terms, approximate inputs by Gaussian with mean μ_α, σ_α (CLT).

$\rightarrow$ For details, visit poster BP 53.10, today from 5 to 7 pm or consult Dahmen et al., arXiv:1512.01073.

Population average



- $m_\alpha(t)$ measures average activity in population α.

$$m_\alpha(t) := \frac{1}{N_\alpha} \sum_{i \in \alpha} \langle n_i(t) \rangle = \overline{m}_\alpha + \delta m_\alpha(t) \qquad (1)$$

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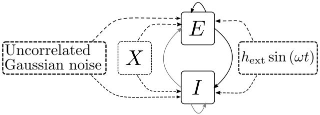

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- $c_{\alpha\beta}(t)$ measures excess synchronous activity compared to the assumption of independence.

$$\begin{aligned} c_{\alpha\beta}(t) &:= \frac{1}{N_\alpha N_\beta}\sum_{i\in\alpha, j\in\beta, i\neq j}\langle n_i(t)\, n_j(t)\rangle - \langle n_i(t)\rangle\langle n_j(t)\rangle \\ &= \overline{c}_{\alpha\beta} + \delta c_{\alpha\beta}(t),\ \alpha,\beta\in\{E, I, X\} \end{aligned} \qquad (2)$$

Deviation of mean activity from stationarity in Fourier space

$$\tau \frac{\partial}{\partial t} \delta \boldsymbol{m}\left(\boldsymbol{t}\right) + \delta \boldsymbol{m}\left(\boldsymbol{t}\right) \approx W \delta \boldsymbol{m}\left(t\right) + \overline{\boldsymbol{S}} h_{\text{ext}} \sin\left(\omega t\right), \qquad (3)$$

with susceptibility $\overline{S}_\alpha$ and effective connectivity matrix $W_{\alpha\beta}$

$$\overline{S}_\alpha := \frac{1}{\sqrt{2\pi}\overline{\sigma}_\alpha} e^{-\frac{(\overline{\mu}_\alpha - \theta_\alpha)^2}{2(\overline{\sigma}_\alpha)^2}}, \; W_{\alpha\beta} := \overline{S}_\alpha K_{\alpha\beta} J_{\alpha\beta}. \qquad (4)$$

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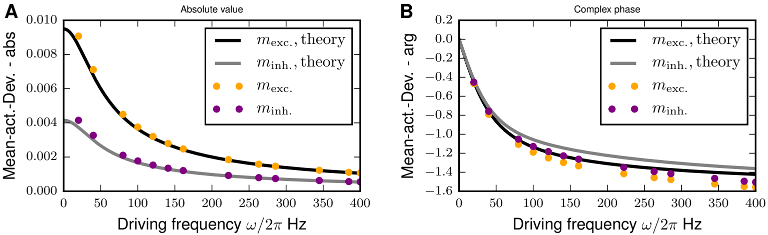
Ansatz $\delta m_\alpha\left(t\right)=M_\alpha^1 e^{i\omega t}$ leads to

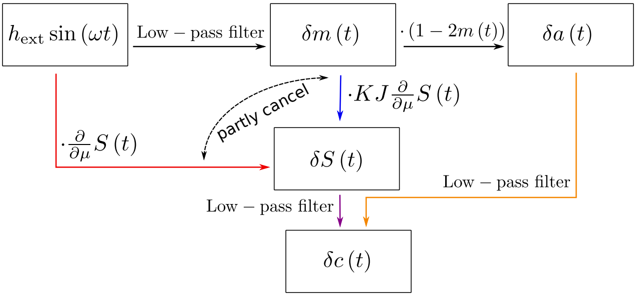
$$M_\alpha^1 = U_{\alpha\beta}\frac{h_{\mathrm{ext}}\left(U^{-1}\overline{S}\right)^\beta\left(-i\tau\omega+1-\lambda^\beta\right)}{\left(\tau\omega\right)^2+\left(1-\lambda^\beta\right)^2}. \qquad (5)$$

Mean activities in two populations



- The mean activities decay like $\frac{1}{\omega\tau}$ for large driving frequency ω.
- Low-pass-filter-"cutoff" $\omega_{\mathrm{cut}}\tau$ given by eigenvalues of the connectivity matrix W.

Derivation of the ODE for $\delta c\,(t)$

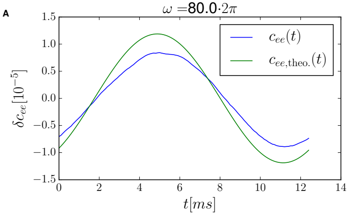


$$\tau \frac{\partial}{\partial t}\delta m_\alpha(t) + \delta m_\alpha(t) \approx \sum_\beta W_{\alpha\beta}\delta m_\beta(t) + \overline{S}_\alpha h_{\text{ext}} \sin(\omega t)$$

$$\mu(t) = KJm(t) + h_{\text{ext}} \sin(\omega t)$$

$$\tau \frac{d}{dt} c_{\alpha\beta}(t) + 2\cdot c_{\alpha\beta}(t) = \left\{ \sum_\gamma S(\mu_\alpha(t), \sigma_\alpha) K_{\alpha\gamma} J_{\alpha\gamma} \left(c_{\gamma\beta}(t) + \delta_{\gamma\beta} \frac{a_\beta(t)}{N_\beta} \right) \right\} + \{\alpha \leftrightarrow \beta\}$$

Deviation of the correlation from stationarity



$$\delta c\,(t) = \mathrm{Im}\left(C_1\,(\omega)\,e^{i\omega t}\right)$$

For the parameters used here, the modulation via S dominates the dependence of $C_1\,(\omega)$ on ω.

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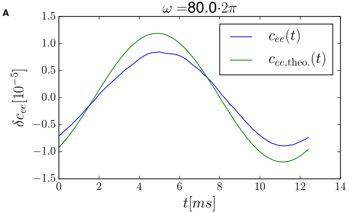

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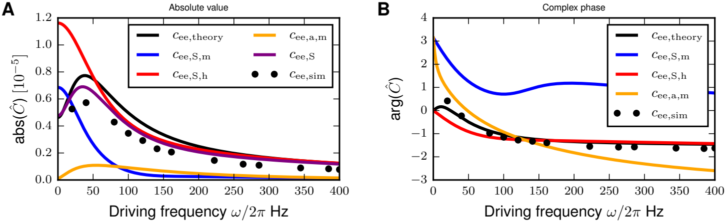

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  - modulation of intrinsic fluctuations $\propto a(t) \propto m(t)(1 - m(t))$
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- Therefore, time modulated correlations observed in neuronal networks can qualitatively be understood and their mechanisms explained.
- However, further investigation of the adaptation to real experimental setups and of structured networks are necessary.

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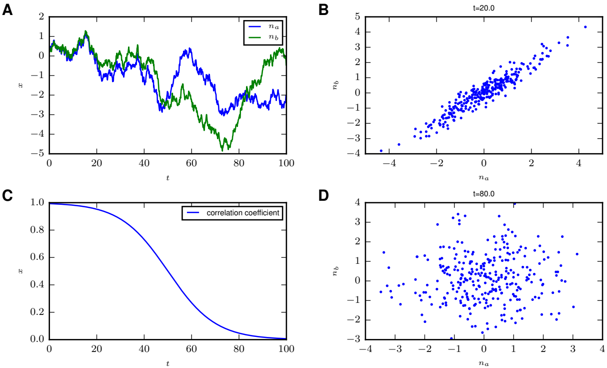
Thank you for your attention!

Appendix: Time-dependent pairwise correlations



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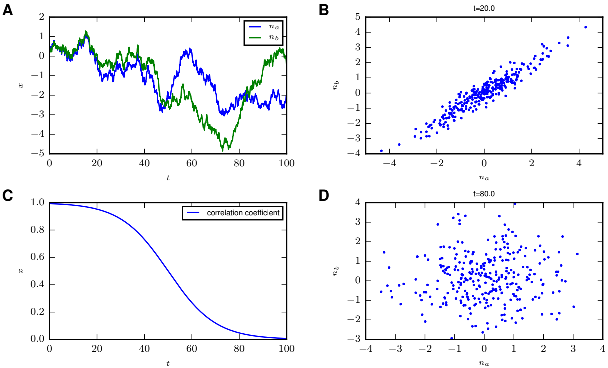

- Signals $n_a,\ n_b$, both with mean 0, noise signals $y_a,\ y_b,\ y_c$ drawn from independent Ornstein-Uhlenbeck processes

$$n_i = \sqrt{c(t)}\,y_c + \sqrt{1-c(t)}\,y_i,\quad i \in \{a, b\} \qquad (6)$$

$$W_{\alpha\beta} := \overline{S}_\alpha K_{\alpha,\beta} J_{\alpha,\beta},\ V_{\alpha\beta} := \frac{\Theta_\alpha - \overline{\mu}_\alpha}{(\overline{\sigma}_\alpha)^2} W_{\alpha\beta},\ \ T_{\alpha\beta} = K_{\alpha\beta} J_{\alpha\beta} \quad (7)$$

$$\begin{aligned}
&\tau \frac{d}{dt} \delta c(t) + \{(\mathbb{1} - W)\, \delta c(t)\} + \{\,..\}^T \\
&= (T \delta m(t))^{\mathrm{diag}}\, V \left(\overline{c} + \overline{a}^{\mathrm{diag}}\right) \rightarrow \text{Variation of } S \text{ via } \mathbf{m}. \\
&+ h_{\mathrm{ext}} \sin(\omega t)\, V \left(\overline{c} + \overline{a}^{\mathrm{diag}}\right) \rightarrow \text{Variation of } S \text{ via } \mathbf{h}_{\mathrm{ext}} \sin(\omega t)\,. \\
&+ W \left(\mathbb{1} - 2\overline{m}^{\mathrm{diag}}\right) (\delta m(t))^{\mathrm{diag}} \Big\} \rightarrow \text{Variation of } a \text{ via } \mathbf{m}. \\
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\end{aligned} \quad (8)$$

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$$+ \{...\}^T\,, \quad (8)$$

For small ω, direct contribution has opposite sign than the contribution from the recurrent, effectively inhibitory feedback.

Experimental background - Setting

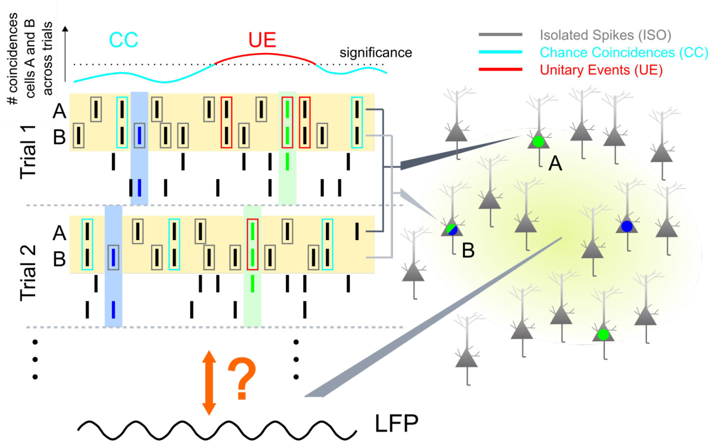
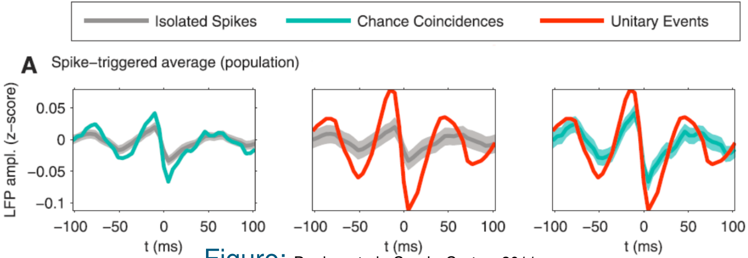


Figure: Denker et al. 2011

Experimental motivation - Locking to LFP-phases



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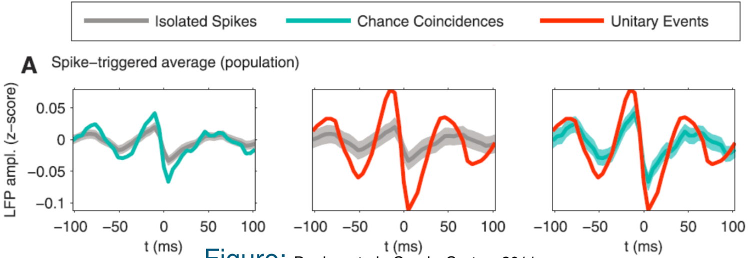

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